On linear extrapolation of observational data of Solar System bodies to the early stages of their formation

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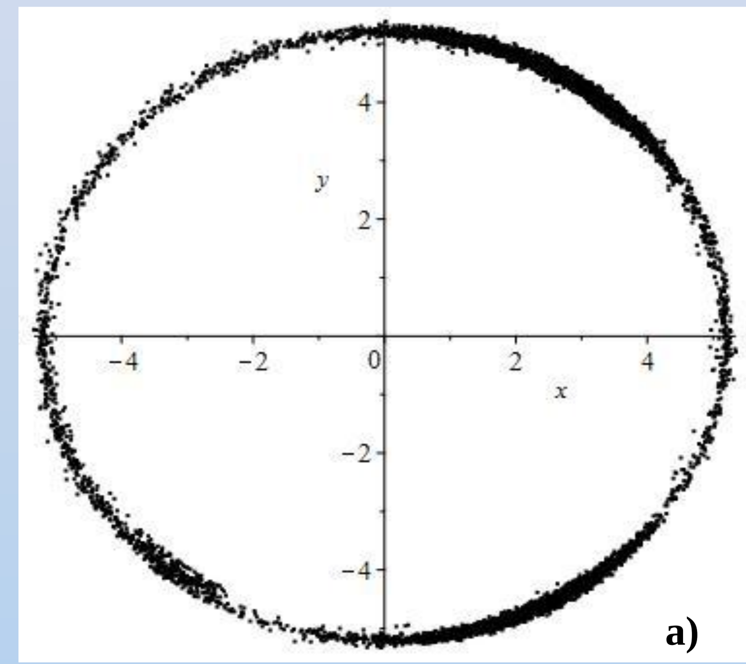
ABSTRACT. In the present paper: 1) the approximate value of the rate of influx of dust particles onto the equatorial disk and its ring-shaped fragments determined using estimates of the initial moment of formation of neighboring fragments of the equatorial disk; 2) using estimates of the critical value of the resonance parameter and the initial moments of the formation of protoplanetary rings, the dynamic viscosity of turbulent motion in these rings is obtained.

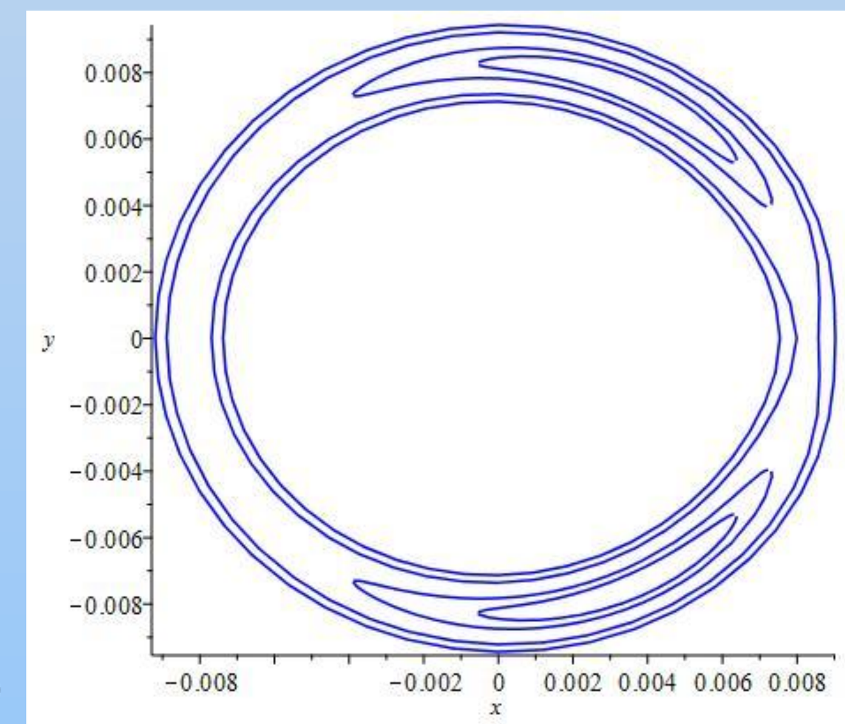
Introduction. In 1969, Safronov's monograph was published [1], which presented the results of a study of the evolution of the preplanetary cloud. In 1971 by Garfinkel, Jupp and Williams [2] an ideal resonance model was developed. Based on this model, in a number of works [3-5], B. Garfinkel developed a theory of the motion of asteroids near the 1/1 orbital resonance, a theory of the motion of Trojan asteroids. Libration motions of asteroids in orbital resonance 1/1 are stable near the Lagrange points L_4 and L_5 to small perturbations in the radial direction. In this case, the magnitude of the perturbations based on the true orbital anomaly will not be limited, and it is possible to determine an analogue of the Gaussian ring for long-period librations. In the case of the distribution of the mass m of Jupiter along its orbit according to the law of proportionality $\Delta m = m\cdot\Delta t/T$, introduced by Gauss, we obtain a Gaussian ring if the value Δt is determined using Kepler's second law. If Δt is calculated by solving the ideal resonance problem [4], then we obtain a similar distribution for long-period librations. In a ring with a distribution for long-period librations, Lagrange points will exist in the gas-dust disk before the appearance of Jupiter. As a result of turbulent motion in these disks, dust particles will accumulate at stable Lagrange points L_4 and L_5, forming dense bodies of any size. That is, with the help of such turbulence it is possible to solve the problem of the formation and growth of celestial bodies up to a meter in size or more. Therefore, it is necessary to determine the dynamic viscosity, which can be associated with turbulent movement and formation of bodies. To solve this problem, in this work, using estimates of the critical value of the resonance parameter and the initial moments of the formation of protoplanetary rings, the dynamic viscosity of turbulent motion in these rings is obtained.

Quasi-periodic motion of small bodies in gas and dust disks of young stars. Information about the stages of convective compression can be stored by a gas-dust disk formed on the equatorial plane. In this case, it is necessary to determine the main mechanisms influencing and determining the redistribution of the dust component in gas-dust disks. One of these mechanisms, at the initial stage of the evolution of gas and dust disks, may be the mechanism of density waves [8], which will inevitably form, during the accretion of gas onto the core of a protostar, acoustic waves. The main, first, wave resonator is the core of the protostellar cloud, and the second will be the gas and dust shell. The movements of density waves in the shell of the protostar will move dust particles, forming ring-shaped fragments inside the gas-dust disk [8]. The most difficult problem is the process of the initial formation of bodies and the mechanism of growth of the masses of these bodies. Is this process the result of random collisions or are celestial bodies formed under the influence of regular mechanisms? Simulation of gas dynamics inside the disk using computer systems (stationary mode) [7] shows that in the case of a constant influx of gas into the disk, gas circulation zones will form inside the disk, reminiscent of the libration orbits of Trojan group asteroids. The libration orbits of the Trojans are formed near two stable Lagrange points L_4 and L_5. Therefore, in the case of co-orbital asteroids of Jupiter, there will be two circulation zones. The restricted three-body problem considers the motion of a body of almost zero mass. The masses of asteroids are small compared to the mass of the Sun and the mass of Jupiter. The motion of each co-orbital asteroid of Jupiter is considered separately and independently of the motion of other asteroids. Their orbits are similar in many ways: the orbits of all these asteroids are near the orbit of Jupiter (data from the TMP IAU catalog, Fig. 1a), and many of them have almost identical libration orbits. In this sense, their movement is similar to a stream of meteoroids. Consequently, co-orbital asteroids of Jupiter are best suited for testing the model of body formation and studying the processes of the initial formation of bodies in gas-dust disks. Small perturbations in the density of a gas-dust disk can change the direction of the velocity vector of Keplerian particle orbits and dynamic viscosity. The changed dynamic viscosity can be determined using the Navier–Stokes equation.

For intermediate values of α from α = 0 to the critical value, libration orbits will have the form shown in Fig. 1, b. According to the ideal resonance model, small bodies that fall into the Lagrange points L_4 and L_5 will remain at these points indefinitely if there is no influence of external forces. Let us assume that in the case of the influence of external causes, the resonance parameter will be some function of time with a minimum zero value and a maximum critical value. Such an external cause in the early stages of body formation may be the inhibition of dust particles settling on the equatorial plane of the protostar. Taking this into account, let us represent the resonance parameter as a linear function $\alpha(t) = \alpha_0 \pm \alpha_1 (t - t_0)$. Here α_0 is a constant that is determined in the ideal resonance model, and the constant coefficient α_1 characterizes the intensity of the influx of dust particles onto the equatorial disk. The approximate value of the rate of influx of dust particles onto the equatorial disk and its ring-shaped fragments can be determined using estimates of the initial moment of formation of neighboring fragments of the equatorial disk [6]. The beginning of the formation of the inner, neighboring ring will be considered the time of the end of the influx of dust particles from the shell of the protostar to the ring in question. The initial moment is the beginning of the formation of this ring. Taking this into account, using the formula $\alpha_c = \alpha_0 + \alpha_1 (t_{i+1} - t_i)$ we obtain: $\alpha_c / \alpha_1 = t_{i+1} - t_i$, where αc is the critical value of the resonance parameter, $\alpha_0 = 0$. The critical value $\alpha_c = 1.41$ is determined using the results modeling long-period librations [8]. Therefore, the coefficient α_1 will be known: $\alpha_1 = \alpha_c /(t_{i+1} - t_i)$. Taking into account the values of ti obtained in [6], we determine the value of the coefficient α1 for the protoplanetary rings of the early Solar System. For the ring of Neptune $\alpha_1 = 0.699$, for the ring of Uranus $\alpha_1 = 3.84$, for Pluto $\alpha_1 = 0.148$, Saturn $\alpha_1 = 0.698$, Jupiter $\alpha_1 = 0.437$, Ceres (asteroid ring) $\alpha_1 = 0.834$, Mars $\alpha_1 = 5.486$, Mercury $\alpha_1 = 1.53$, Venus $\alpha_1 = 1.55$, Earth $\alpha_1 = 1.54$. Time t in the formula $\alpha_1 = \alpha_c/(t_{i+1} - t_i)$ in million years. The coefficient α1 is taken with a negative sign for the stage of evolution when there is an influx of dust particles into the ring and their accumulation at stable Lagrange points.



a)



b)

Fig. 1. a) Distribution of polar coordinates (r, λ') of 7644 co-orbital asteroids of Jupiter with absolute magnitude *H*, 7 ≤ *H* ≤ 15 from the TMP IAU catalog (Epoch 202005. 31), $r^2 = x^2 + y^2$ – polar radius (AU), λ' – mean synodic longitude of the asteroid; b) libration orbits of Jupiter's co-orbital asteroids for resonance parameters α² = 0.6; 0.8; 1.7; 2.2 with orbital inclinations *i* = 25° in polar coordinates (r^*, λ'), $r^* = r - 0.817$, $r^2 = x^2 + y^2$ (orbital projections onto the plane of Jupiter's orbit).

Dynamic viscosity and the ring density. Dynamic viscosity, taking into account librations, was obtained using the Navier–Stokes equation in the monograph [8] in the following form:

$$\eta(r,z,t) = \frac{3}{4}\left[\frac{gM\rho}{r} - r^{1/4}\frac{\partial\rho}{\partial r}\right] / \left[2mG^3\bar{f}_0' + e\cdot G^2\sin\varphi\left(\frac{1}{(G+\Gamma_0)^3} - \frac{s+q}{s}n_1\right)\right],\quad G = G_0 + G_0^2\Delta_1 + 2G_0^3\Delta_1^2/3 + \ldots, \qquad 1)$$

$$\Delta_1 = -1/3\sqrt{6m}[\alpha^2 - \bar{f}_0(\lambda^*)]^{1/2}\,\mathrm{sgn}(d\lambda^*/dt),\qquad \Delta t = t - t_1 = \frac{1}{\sqrt{6m}G_0^2}\int_{\lambda_1^*}^{\lambda^*}\frac{d\lambda^*}{[\alpha^2 - \bar{f}_0]^{1/2}} - \frac{4}{9}G_0^3(\lambda^* - \lambda_1^*),\quad G_0 = \left[\frac{s}{(s+q)n_1}\right]^{1/3} - \Gamma_0, \qquad 2)$$

where g is the gravitational constant, M is the mass of the Sun (protostar), m is the mass of Jupiter, v, e is the true anomaly and eccentricity of the Keplerian orbit of the asteroid, sin(φ) is a function of time t. The function $f_0(\lambda^*)$ characterizes libration motion [8]. The argument of this function $\lambda^* = \lambda - n_1 (s + q)/s$ is a function of time t and is expressed as a series of Jacobi functions [8], λ, φ, n_1 are the average longitude, the true anomaly of dust particles and the average motion of Jupiter, α is the resonance parameter, s, q are integers that determine the commensurability of the average motions of Jupiter and the dust particle. According to Newton's definition, dynamic viscosity in hydrodynamics is the friction between layers of liquid (or gas). The formation of vortices is a consequence of such friction. In this sense, the dynamic viscosity, which is presented in formula (1), affects the dynamics of gas and dust particles as friction: where the viscosity $\eta(r)$ is maximum, the relative speed of the particles will be minimum and vice versa. As a result of such friction, the mutual distances between dust particles will decrease, which will lead to the accumulation of dust particles and the formation of dense bodies of different sizes. The distribution density according to the law of proportionality $\Delta m = m\cdot\Delta t/T$ for the case of long-period librations will be determined by solving the ideal resonance problem [4]. Applying the value Δt defined in formula (2), we obtain:

$$\Delta m = \frac{m}{T}\Delta t = \frac{m}{T}\frac{1}{\sqrt{6m}G_0^2}\frac{d\lambda^*}{[\alpha^2 - \bar{f}_0]^{1/2}} = \rho_r(r,\lambda^*)\sqrt{r^2 + \left(\frac{dr}{d\lambda^*}\right)^2},\quad T = T_1(\alpha,m)/T_1(0,m) = \frac{3\sqrt{2}}{8\pi}\oint(\alpha_c^2 - f_0)d\lambda^*, \qquad 3)$$

where $T_1(0, m) = 4\pi/(27m)^{1/2}$ is the period of small oscillations near the Lagrange points L_4 and L_5 [4], α_c is the critical value of the resonance parameter α, that is, when $\alpha > \alpha_c$ the asteroid leaves 2) resonance 1 /1, T – libration period of the asteroid. Resolving equation (3) with respect to ρ_r, we obtain the following expression for the ring density for the case of long-period librations:

$$\rho_r(r,\lambda^*) = \frac{m}{T}\frac{1}{\sqrt{6m}G_0^2}\frac{d\lambda^*}{[\alpha^2 - \bar{f}_0]^{1/2}} / \sqrt{r^2 + \left(\frac{dr}{d\lambda^*}\right)^2}$$

According to the formula for density ρ_r, obtained from equality (3), the mass m of Jupiter is distributed along a circular orbit so that the gravitational systems Sun - Jupiter - asteroid and Sun - ring with density ρr are the same. In this case, the Lagrange points L_4 and L_5 of these systems will be the same. In both cases, the points L_4 and L_5 are defined as singular points of the same characteristic function f_0, by which the librations are determined.

Conclusions. observational data from the Solar System seem more suitable for detailing the process of body formation. For this reason, this work examines the distributions of the main ring asteroids and co-orbital asteroids of Jupiter. Using mathematical modeling, an attempt was made to determine the mechanism of the initial formation and growth of the masses of celestial bodies, starting from the size of dust grains to asteroid sizes at stable Lagrange points L_4 and L_5, as well as the mechanism of the formation of ice blocks in the rings of Saturn. As a result of modeling the process of body formation and analysis of observational data, the following conclusions were made in this work: 1) Despite the great successes achieved in methods for modeling the formation of planetary systems, solving the problem of the initial formation of celestial bodies remains the main task. To solve this problem, it is necessary to adequately define the equation of turbulent motion, which is included in the system of gas dynamics equations. An adequate determination of dynamic viscosity and, in general, effective viscosity will make it possible, with the help of modern computer modeling tools, to obtain a holistic picture of evolution from its initial stages to the stage of formation of planetary systems [6]. 2) Simulation of the quasi-periodic motion of Jupiter's co-orbital asteroids shows that the process of formation of small celestial bodies can be divided into two stages: the downward flow stage (minus sign in front of α_1) and the upward flow stage (plus sign in front of α_1).

References

[1] V.S. Safronov, *Evolution of the preplanetary cloud and the formation of the Earth and planets*. M.: Nauka, 1969, 244 p.

[2] B. Garfinkel, A. Jupp A., C. Williams, Astron. J. 76, 157 (1971).

[3] B. Garfinkel, Celest. mech. 13, 229 (1976).

[4] B. Garfinkel, Astron. J. 82, 368 (1977).

[5] B. Garfinkel, Celest. mech. 18, 259 (1978).

[6] T.R. Abdulmyanov, Astrophysics. Bulletin. Vol. 75, No. 2, 132 (2020).

[7] T.R. Abdulmyanov, in: Materials of the II International Scientific Seminar ITON - 2022. (Kazan: KFU Publishing House, 2022), p. 10.

[8] T.R. Abdulmyanov, *Modeling the process of formation of vortex movements in gas and dust disks using computer computing systems* (Kazan: KSEU, 2022), 391 pp.