

Length of day multifractal dynamics

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August 15, 2024

Introduction

As a measure of the speed of the Earth's rotational motion, the duration of the day or LOD (Length of Day) is traditionally used, that is, the deviation of the instantaneous duration of the average solar day from 86400 atomic seconds. It is also known that the LOD time series exhibit a complex structure in which small, but regular, tidal and seasonal fluctuations are superimposed on powerful irregular fluctuations of unknown nature with a characteristic time scale from units to tens of years.

Regular tidal LOD variations are basically due to deformations of the solid Earth by zonal tides. They are well described by simple linear theory of tidal deformation by use of Love numbers. Typical amplitude of tidal variations are at the order of fractions of millisecond. Regular seasonal LOD fluctuations are due to atmospheric circulation, namely to the motion term of the atmospheric angular momentum, that is due to variations of zonal atmospheric winds.

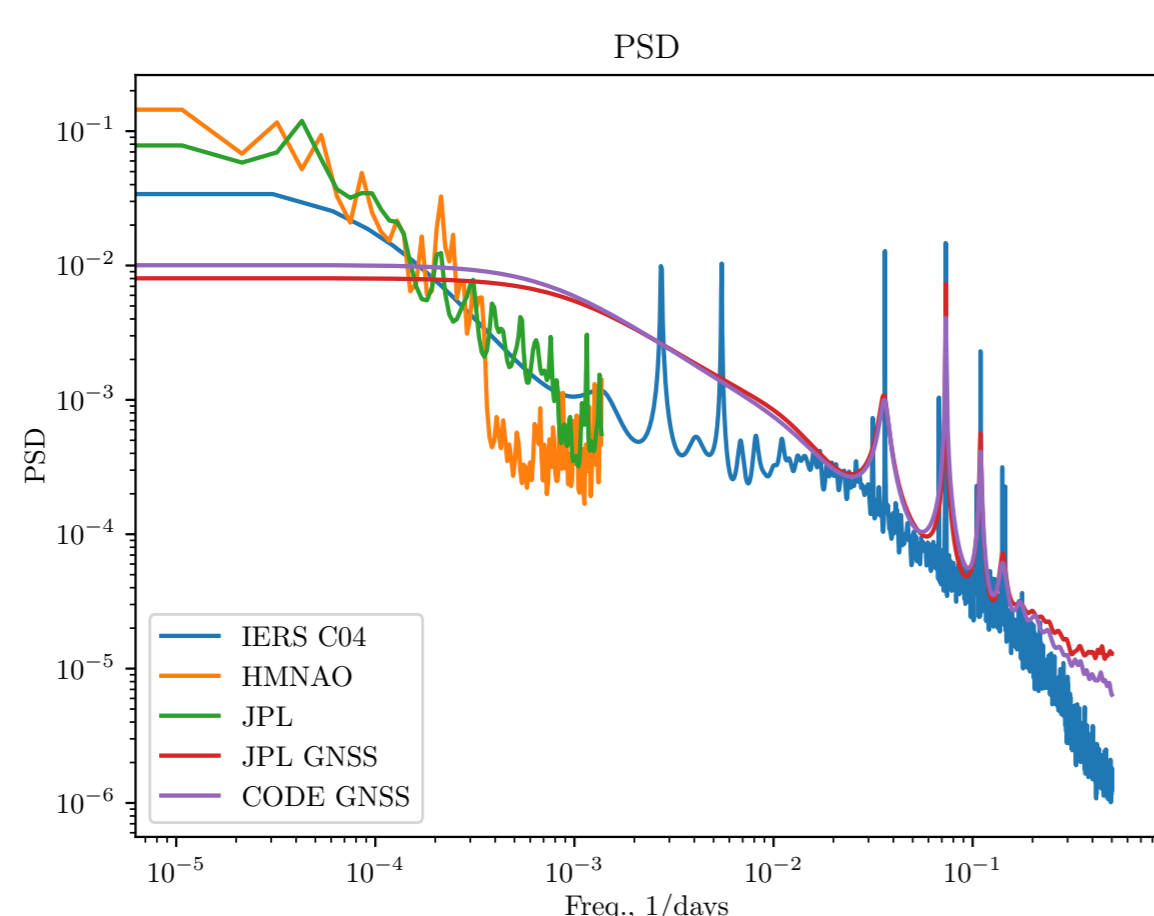


Figure: Power spectrum density for different series

On the other hand, on the regular tidal and seasonal LOD variations huge irregular or chaotic variations of decadal timescale are superimposed. Their nature is still unknown and is probably due to currents in the Earth liquid core or to variations of the Solar wind or to both these effects combined.

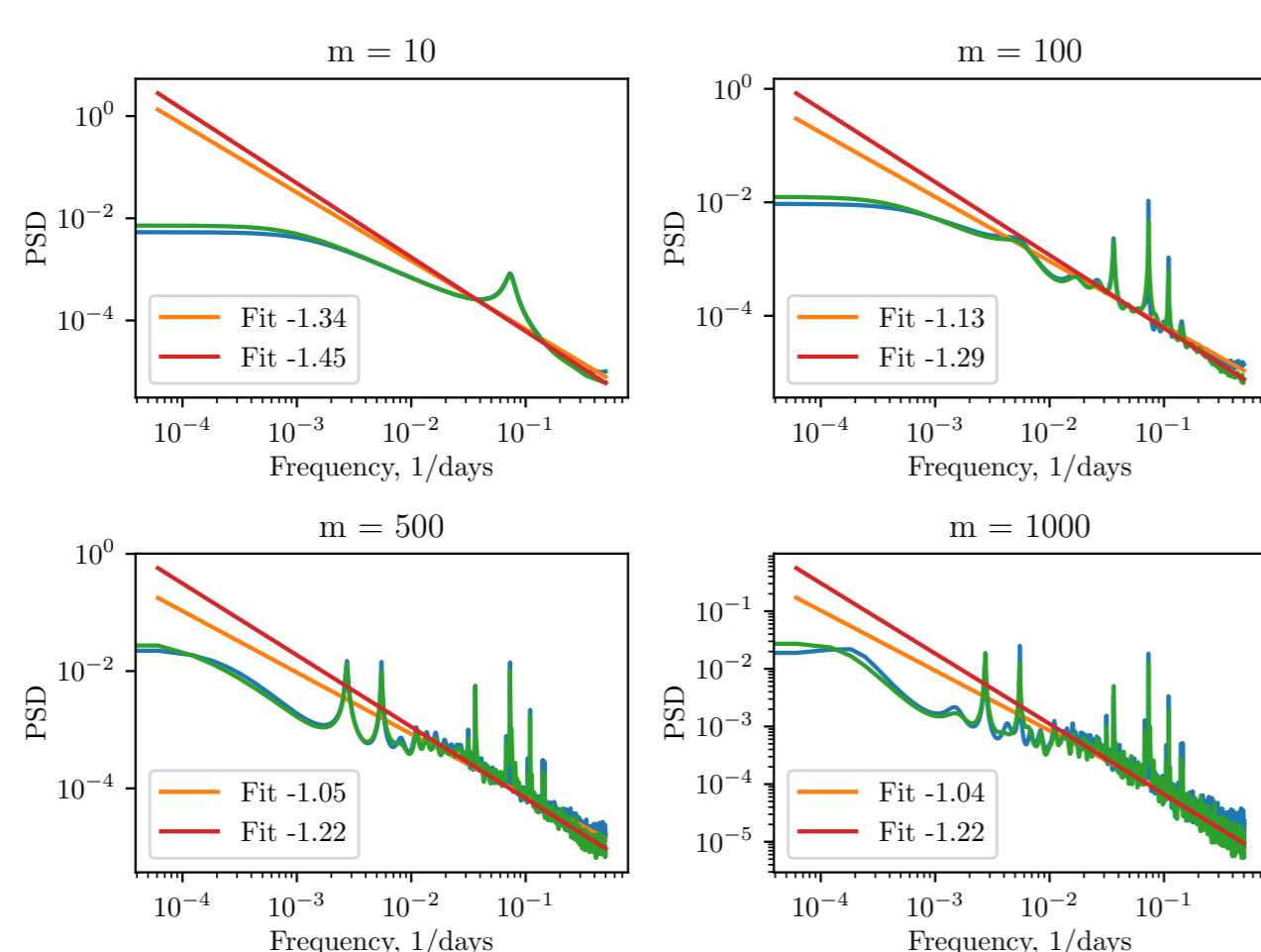


Figure: Linear fit of PSD for different filter length for Burg method, JPL (blue) and CODE (green)

Putting aside the physical nature of decadal LOD fluctuations in this work we concentrate on phenomenological analysis of the LOD time series. Since the appearance of the decadal LOD fluctuations is well nonlinear then it is natural to expect that this phenomenon may exhibit a fractal structure, that is already shown by e.g. [Frede et al.(1999a), Frede et al.(1999b)]. These authors have shown that long term structure of the LOD time series is chaotic and fractal. In this study we concentrate on multifractal structure of the LOD time series by use of their spectral analysis.

Observational data

We selected five different LOD time series: estimates from the combined Earth orientation series LUNAR97 (Jet Propulsion Laboratory, JPL) [Gross et al.(1993)], Measurement of the Earth's rotation from 720 BC to AD 2015 (Her Majesty's Nautical Almanac Office, HMNAO) [Morrison et al.(2021)], Combined Earth Rotation Parameter solution (C04 IERS) and operational series from CODE analysis center [Frede et al.(1999a)]. Of these data HMAO and JPL are the longest ones, continued to the past up to 17th century. The combined IERS C04 series is shorter, up to 19th century in the past. Finally, JPL GNSS and CODE GNSS are the instrumental series based on GNSS data alone. They are the shortest ones and start only about 25 years ago.

It is also important to note that accuracy of these series differ significantly not only from series to series but also may differ for various time periods. Also the longest series are combined from different observations from old and imprecise Lunar occultations to modern and most precise GNSS and VLBI data.

Spectral analysis of LOD series

The well known Burg or Maximum entropy spectral estimation method is used in this study. The only variation of the standard method implemented here is that we do not limit the filter length but instead allow it to be as big as needed for a decent spectral resolution.

In Figure 1 the power spectral densities for different LOD time series are shown in logarithmic scale on both axes. It can be seen from the Figure that different LOD time series reveal similar power spectra. Differences in spectra may be due to different observational data used and degree of smoothing in the series.

In Figure 2 power spectra for GNSS series are shown for different value of filter length in Burg method. It can be seen that values of the exponents fitted converge well with increase of filter length. However, JPL and CODE series give slightly different fitted exponents. This effect shall be subject to further analysis.

Summary

A stable multifractal structure, practically independent of the type of time series, was revealed in the resulting power spectrum. The power spectrum of the duration of the day can be divided into three frequency ranges. For periods of less than three months, LOD behaves like a power noise with an indicator close to -1.5. For periods from three months to three years, LOD behaves almost like white noise. Finally, for periods of more than three years, LOD also demonstrates the behavior of power noise with an index of about -1.5. It turns out that the revealed structure practically does not depend on the type of time series (combined or instrumental), its length, as well as on the parameters of the spectral power estimation method. The paper also presents the first attempt to build a dynamic model of the identified multifractal LOD structure, which can be used both to predict the duration of the day and for the purpose of physical interpretation of variations in the speed of rotational motion of the Earth.

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