DIRAC PARTICLE IN GRAVITATIONAL FIELD OF BLACK HOLE WITH NEWMAN-UNTI-TAMBURINO PARAMETER

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INTRODUCTION

Recently, the metrics with Newman-Unti-Tamburino (NUT) parameter have attracted the interest of scientific community. NUT parameter is usually interpreted as a gravitomagnetic charge (monopole) or as a linear source of a pure angular momentum (the twisting of the surrounding spacetime) [Chakraborty et al. (2023); Galtsov et al. (2017)]. So, the black holes with NUT parameter are considered as physically meaningful systems with some special characteristics. Being axially-symmetric, the NUT black holes can exhibit new effects, such as asymmetry of black hole shadow or as the Lense-Thirring effect demonstrated by the rotating black holes [Chakraborty et al. (2023); Ghasemi-Nodehi et al. (2021)]. The presence of the NUT-parameter in anti-de-Sitter metric leads to the appearance of a region with negative Gibbs free energy in thermodynamics of black holes that is associated to the phase transition Awad et al. (2023). The purpose of this work is to study the quantum-mechanical behavior of fermions in the background of the gravitational field of the NUT black hole.

DIRAC EQUATION IN NUT SPACE-TIME

NUT metric is given as

$$ds^2 = \Phi\left(dt + 4a\sin^2(\frac{\theta}{2})d\phi\right)^2 - \frac{dr^2}{\Phi} - \left(a^2 + r^2\right)\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

where $\Phi = 1 - \frac{r_g r + 2a^2}{r^2 + a^2}$, a is NUT parameter; when $a = 0$, the NUT metric reduces to the Schwarzschild one.

The general covariant Dirac equation

$$\left[i\gamma^a\left(e^{\alpha}_{(a)}\frac{\partial}{\partial x^\alpha} + \frac{1}{2}\sigma^{mn}\gamma_{mna}\right) - M\right]\Psi = 0$$

takes the form

$$\Big[i\Big(\gamma^0\frac{\rho}{\sqrt{\Delta}} + \gamma^3\frac{2a}{\rho}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\Big)\frac{\partial}{\partial t} - i\gamma^1\Big(\frac{\sqrt{\Delta}}{\rho}\frac{\partial}{\partial r} + \frac{r\sqrt{\Delta}}{2\rho^3} + \frac{\Delta'}{4\rho\sqrt{\Delta}}\Big)$$
$$+ i\gamma^0\gamma^2\gamma^3\frac{a\sqrt{\Delta}}{2\rho^3} - i\gamma^2\frac{1}{\rho}\Big(\frac{\partial}{\partial\theta} + \frac{1}{2\tan\theta}\Big) - i\gamma^3\frac{1}{\rho}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} - M\Big]\Psi = 0.$$

Here we introduce $\rho^2 = r^2 + a^2$, $\Delta = r^2 - r_g r - a^2$, $\Phi = \frac{\Delta}{\rho^2}$ and assume the Weyl basis

$$\Psi = \begin{vmatrix}\xi \\ \chi\end{vmatrix}, \quad \gamma^0 = \begin{vmatrix}0 & I \\ I & 0\end{vmatrix}, \gamma^1 = \begin{vmatrix}0 & -\sigma_3 \\ \sigma_3 & 0\end{vmatrix}, \gamma^2 = \begin{vmatrix}0 & -\sigma_1 \\ \sigma_1 & 0\end{vmatrix}, \gamma^3 = \begin{vmatrix}0 & -\sigma_2 \\ \sigma_2 & 0\end{vmatrix}.$$

We derive equations in 2-spinor form

$$\sigma_1\Big(\frac{1}{\rho}\chi_{,2} + \frac{1}{2\rho\tan\theta}\chi\Big) + \sigma_2\Big(\frac{1}{\rho\sin\theta}\chi_{,3} - \frac{2a}{\rho}\tan\frac{\theta}{2}\,\chi_{,0}\Big)$$
$$+\sigma_3\Big[\frac{\sqrt{\Delta}}{\rho}\chi_{,1} + \Big(\frac{\Delta'}{4\rho\sqrt{\Delta}} + \frac{\sqrt{\Delta}}{2\rho^3}\rho_-\Big)\chi\Big] + \frac{\rho}{\sqrt{\Delta}}\chi_{,0} + iM\xi = 0,$$
$$\sigma_1\Big(\frac{1}{\rho}\xi_{,2} + \frac{1}{2\rho\tan\theta}\xi\Big) + \sigma_2\Big(\frac{1}{\rho\sin\theta}\xi_{,3} - \frac{2a}{\rho}\tan\frac{\theta}{2}\,\xi_{,0}\Big)$$
$$+\sigma_3\Big[\frac{\sqrt{\Delta}}{\rho}\xi_{,1} + \Big(\frac{\Delta'}{4\sqrt{\Delta}\rho} + \frac{\sqrt{\Delta}}{2\rho^3}\rho_+\Big)\xi\Big] - \frac{\rho}{\sqrt{\Delta}}\xi_{,0} - iM\chi = 0;$$

where we apply the notations

$$\partial_\alpha =_{,\alpha},\ \rho_+ = r + ia,\ \rho_- = r - ia.$$

We search spinors in the form

$$\xi = \Delta^{-1/4}\rho_+^{-1/2}e^{-i\epsilon t}e^{im\phi}X(r,\theta),\ \chi = \Delta^{-1/4}\rho_-^{-1/2}e^{-i\epsilon t}e^{im\phi}Y(r,\theta),$$

the variables are separated within the substitutions

$$X_1 = R_1(r)T_1(\theta),\ X_2 = R_2(r)T_2(\theta),$$
$$Y_1 = R_3(r)T_1(\theta),\ Y_2 = R_4(r)T_2(\theta).$$

SUMMARY

We may expect that the present analysis can permit us to elucidate the physical interpretation for the Newman-Unti-Tamburino metric. The question arises about possible existence of the Hawking-like radiation in this model.

ANGULAR EQUATIONS

The angular equations are

$$\frac{dT_1}{d\theta} + \Big(\frac{1}{2\tan\theta} - \frac{m}{\sin\theta} - 2a\epsilon\tan\frac{\theta}{2}\Big)T_1 - \Lambda T_2 = 0,$$
$$\frac{dT_2}{d\theta} + \Big(\frac{1}{2\tan\theta} + \frac{m}{\sin\theta} + 2a\epsilon\tan\frac{\theta}{2}\Big)T_2 - \Lambda T_1 = 0;$$

solutions are constructed in the hypergeometric functions. The quantization rule for the separation parameter Λ is

$$\Lambda^2 = -(m + n_1 + 1/2)(m + n_1 + 1/2 + 4a\epsilon),\ m > 0.$$

RADIAL EQUATIONS

The radial equation system is

$$\Big(\sqrt{\Delta}\frac{d}{dr} - \frac{ia\sqrt{\Delta}}{\rho^2} - \frac{i\epsilon\rho^2}{\sqrt{\Delta}}\Big)R_3 + iM\rho_- R_1 = \Lambda R_4,$$
$$\Big(\sqrt{\Delta}\frac{d}{dr} + \frac{ia\sqrt{\Delta}}{\rho^2} + \frac{i\epsilon\rho^2}{\sqrt{\Delta}}\Big)R_1 - iM\rho_+ R_3 = \Lambda R_2,$$
$$\Big(\sqrt{\Delta}\frac{d}{dr} - \frac{ia\sqrt{\Delta}}{\rho^2} + \frac{i\epsilon\rho^2}{\sqrt{\Delta}}\Big)R_4 - iM\rho_- R_2 = \Lambda R_3,$$
$$\Big(\sqrt{\Delta}\frac{d}{dr} + \frac{ia\sqrt{\Delta}}{\rho^2} - \frac{i\epsilon\rho^2}{\sqrt{\Delta}}\Big)R_2 + iM\rho_+ R_4 = \Lambda R_1.$$

In massless case, we found solution in the terms of the confluent Heun functions $H(v)$, $v = \frac{r - r_2}{r_1 - r_2}$, as follows

$$R_1 = \frac{\epsilon^{1+i\epsilon r_g}e^{i\epsilon(r-r_1)}\left(r - r_1\right)^{\frac{1}{2}+i\epsilon r_1}\left(r - r_2\right)^{\frac{1}{2}+i\epsilon r_2}\sqrt{r + ia}}{\sqrt{r - ia}}H(v).$$

EFFECTIVE POTENTIAL

In order to compare the present study with the results in Schwarzschild case, we introduce variables $f = R_1 + R_2$, $g = i(R_1 - R_2)$ and turtle-like coordinate w, then in massless case the radial equations have the structure

$$\Big[\frac{d^2}{dw^2} + P(w)\Big]f = 0,$$

where $P(w)$ is considered as effective potential

$$P = \frac{\left(x - x_1\right)^2\left(x - x_2\right)^2}{(a - ix)^4}\Big(\frac{\left(a^2 + x^2\right)^2}{\Delta^2} + \frac{ia\Delta'}{\Delta\left(a^2 + x^2\right)} +$$
$$+ \frac{2\Lambda x}{\sqrt{\Delta}\left(a^2 + x^2\right)} - \frac{a(a + 4ix)}{\left(a^2 + x^2\right)^2} - \frac{\Lambda\Delta'}{2\Delta^{3/2}} - \frac{\Lambda^2}{\Delta}\Big)$$